

Area, circles and volume

Topic cliff notes / objectives, key ideas and examples

Learning objectives

- Choose area and circumference formulas.
- Calculate prism volume and give the correct units.

Key ideas

- Use a perpendicular height for triangles and parallelograms.

$$A_{\text{triangle}} = \frac{1}{2}bh, \quad A_{\text{parallelogram}} = bh$$

- For a trapezium, use the two parallel sides and perpendicular height.

$$A = \frac{1}{2}(a + b)h$$

- Distinguish circle area from circumference; radius is half the diameter.

$$A = \pi r^2, \quad C = 2\pi r = \pi d$$

- A prism has a constant cross-section.

$$V = \text{cross-sectional area} \times \text{length}; \quad V_{\text{cylinder}} = \pi r^2 h$$

- A sector is a fraction of a complete circle.

$$\text{sector fraction} = \frac{\theta}{360^\circ}$$

Worked example

Find the exact volume of a cylinder with the given radius and height.

$$r = 3 \text{ cm}, \quad h = 8 \text{ cm} : \quad A = \pi(3)^2 = 9\pi \text{ cm}^2$$

$$V = 9\pi \times 8 = 72\pi \text{ cm}^3$$

Answer

$$72\pi \text{ cm}^3$$

Quick example

Find the trapezium area from its parallel sides and perpendicular height.

$$a = 5, \quad b = 9, \quad h = 4 \text{ (cm)} : \quad A = \frac{1}{2}(5 + 9)4 = 28 \text{ cm}^2$$

Answer

$$28 \text{ cm}^2$$

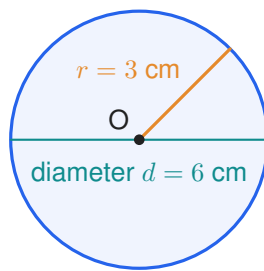
Watch out

Use the radius in the circle area formula. Area uses square units; volume uses cubic units.



Visual guide: circle measures and area

A circle has two different measures. Circumference is the distance around its boundary. Area measures the region inside it. The radius goes from the centre to the circle; the diameter passes through the centre.



Boundary: circumference

$$C = 2\pi r = 6\pi \text{ cm}$$

Inside: area

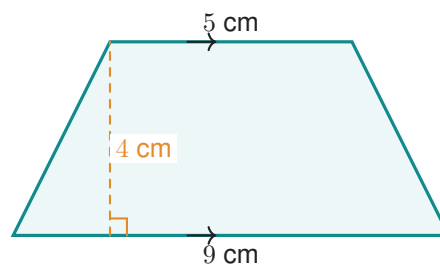
$$A = \pi r^2 = 9\pi \text{ cm}^2$$

$$d = 2r \quad \text{and} \quad r = \frac{d}{2}$$

If a diameter is given, halve it before using $A = \pi r^2$. Keep π in an exact answer, or round only at the end if a decimal is requested.

A trapezium uses its perpendicular height

The parallel sides have lengths 5 cm and 9 cm. The dashed height meets the base at a right angle; the sloping sides are not the height.



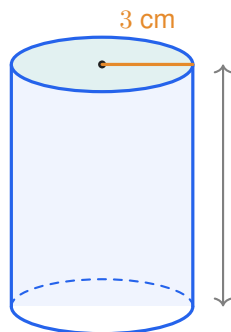
$$A = \frac{1}{2}(a + b)h = \frac{1}{2}(5 + 9) \times 4 = 28 \text{ cm}^2$$

This is the average of the two parallel lengths, multiplied by the perpendicular height. **Area uses square units.**



Visual guide: constant cross-sections

A prism keeps the same cross-section along its length. A cylinder has a circular cross-section. First find the cross-sectional area, then multiply by the length or height.

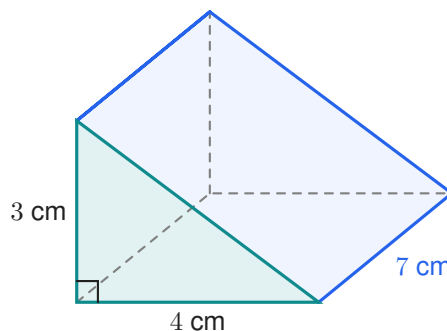
Cylinder: radius 3 cm, height 8 cm**1. Circular cross-section**

$$A = \pi \times 3^2 = 9\pi \text{ cm}^2$$

2. Multiply by height

$$V = 9\pi \times 8 = 72\pi \text{ cm}^3$$

The radius is the orange line from the centre, not the whole width. The dashed arc is the hidden rear edge. The picture shows the cylinder in perspective.

Triangular prism: keep the cross-section separate

The green end is a right-angled triangle. The 7 cm measurement runs *along* the prism, between matching vertices. Perspective drawing; use the labelled lengths.

$$A_{\text{triangle}} = \frac{1}{2} \times 4 \times 3 = 6 \text{ cm}^2$$

$$V = 6 \times 7 = 42 \text{ cm}^3$$

Volume uses cubic units. A sloping side of the cross-section is not its perpendicular height.

