

Pythagoras and right-angled trigonometry

Topic cliff notes / objectives, key ideas and examples

Learning objectives

- Calculate a missing side in a right-angled triangle.
- Choose a trigonometric ratio for a side or angle.

Key ideas

- The hypotenuse is opposite the right angle. Pythagoras applies only to right-angled triangles.

$$a^2 + b^2 = c^2 \quad (\text{hypotenuse } c)$$

- Add squares to find the hypotenuse; subtract squares to find a shorter side. Then take a square root.
- Label opposite and adjacent relative to the chosen acute angle.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

- Use the tangent ratio when the hypotenuse is not involved.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

- For a missing angle, use the appropriate inverse ratio in degree mode.

Worked example

Find the unknown shorter side of a right-angled triangle.

$$c = 13 \text{ cm}, \quad a = 5 \text{ cm} : \quad x^2 + 5^2 = 13^2$$

$$x^2 = 169 - 25 = 144$$

A length takes the positive root.

$$x = \sqrt{144} = 12 \text{ cm}$$

Answer

12 cm

Quick example

Find the side opposite the acute angle in a right-angled triangle.

$$\theta = 30^\circ, \quad c = 10 \text{ cm} : \quad \sin 30^\circ = \frac{o}{10}$$

$$o = 10 \sin 30^\circ = 10 \times \frac{1}{2} = 5 \text{ cm}$$

Answer

5 cm

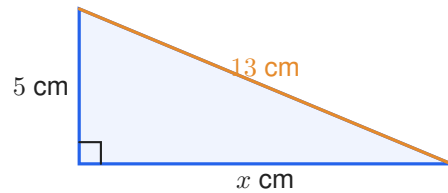
Watch out

A squared length is not the final length. Take the square root and check that the hypotenuse is the longest side.



Visual guide: identify the right angle first

The small square marks a right angle. The side opposite that square is the **hypotenuse**, the longest side of a right-angled triangle.

Pythagoras: a missing shorter side

The unknown side is *not* the hypotenuse. Rearrange by subtracting the known shorter side's square:

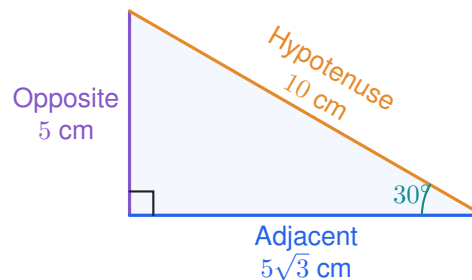
$$x^2 + 5^2 = 13^2 \implies x^2 = 169 - 25 = 144$$

$$x = \sqrt{144} = 12 \text{ cm}$$

Check: $5^2 + 12^2 = 13^2$, and 13 cm is the longest side. A length takes the positive root.

Trigonometry: label sides from the chosen angle

Here the chosen angle is 30° . Opposite and adjacent refer to *that angle*; the hypotenuse still sits opposite the right angle.



$$\sin 30^\circ = \frac{\text{opposite}}{10} \implies \text{opposite} = 10 \sin 30^\circ = 5 \text{ cm}$$

$$\cos 30^\circ = \frac{\text{adjacent}}{10} \implies \text{adjacent} = 10 \cos 30^\circ = 5\sqrt{3} \text{ cm} \approx 8.66 \text{ cm}$$

Choose the ratio using the two sides involved. Use degree mode for these angles. The labels are consistent: $5^2 + (5\sqrt{3})^2 = 25 + 75 = 100 = 10^2$.

