

Transformations, similarity and congruence

Topic cliff notes / objectives, key ideas and examples

Learning objectives

- Describe and perform a transformation completely.
- Distinguish similarity from congruence.

Key ideas

- A translation specifies horizontal and vertical movements, often as a column vector.
- A reflection needs a mirror line; a rotation needs a centre, angle and direction.
- An enlargement needs a centre and scale factor. For a positive factor, multiply distances from the centre.
- Similar figures have equal corresponding angles and a common corresponding length ratio.
- Congruent figures have equal corresponding lengths and angles. Use a valid triangle congruence test.

SSS, SAS, ASA, RHS

Worked example

Enlarge the point about the origin by the given positive factor.

$$A = (2, 3), \quad O = (0, 0), \quad k = 2$$

Multiply both distances from the origin.

$$A' = (2 \times 2, 2 \times 3) = (4, 6)$$

Answer

$$A' = (4, 6)$$

Quick example

Reflect the point in the vertical axis. Reverse the horizontal coordinate; keep the vertical coordinate.

$$(3, -2) \xrightarrow{\text{reflection in } y\text{-axis}} (-3, -2)$$

Answer

$$(-3, -2)$$

Watch out

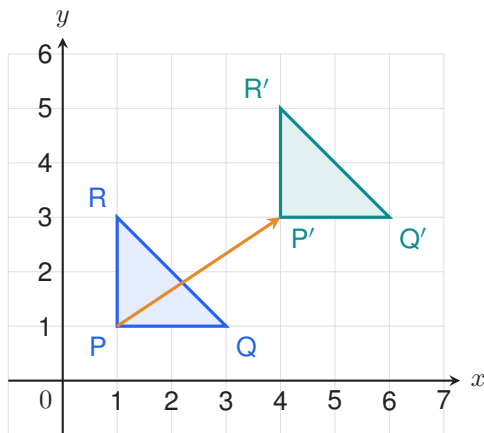
Equal corresponding angles establish triangle similarity, not congruence. An enlargement acts on distances from its centre.



Visual guide: translation and reflection

The blue triangle is the original shape; the green triangle is its image. Each square on the coordinate grids represents one unit.

Translation: slide every point by the same vector



Move **3 right and 2 up**:

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

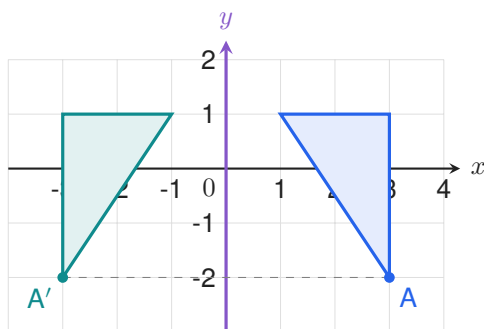
$$(x, y) \mapsto (x + 3, y + 2)$$

For example,

$$P(1, 1) \mapsto P'(4, 3)$$

Q(3, 1) moves to Q'(6, 3), and R(1, 3) to R'(4, 5). The orientation and lengths stay the same.

Reflection: mirror in the y-axis



The purple line is the mirror line $x = 0$. Matching points have the same perpendicular distance from it, on opposite sides.

$$(x, y) \mapsto (-x, y)$$

$$A(3, -2) \mapsto A'(-3, -2)$$

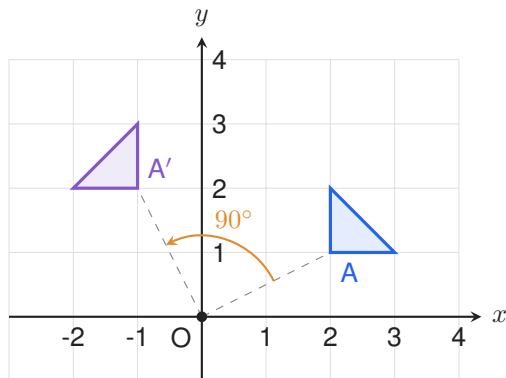
Only the sign of x changes. The reflected shape has the same side lengths and angles.

A complete description names the transformation: for a translation, give its vector; for a reflection, give the mirror line.



Visual guide: rotation and enlargement

Rotation: centre, angle and direction



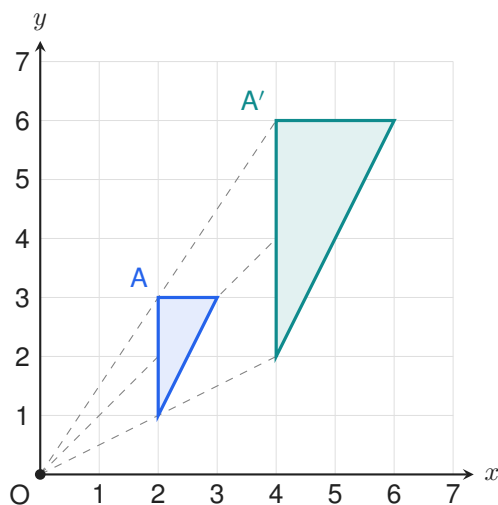
Rotate 90° **anticlockwise** about the origin O. The orange arrow shows the direction.

$$(x, y) \mapsto (-y, x)$$

$$A(2, 1) \mapsto A'(-1, 2)$$

The other vertices move from (3, 1) to (-1, 3) and from (2, 2) to (-2, 2). Each distance from O stays the same.

Enlargement: multiply distances from the centre



Enlarge by scale factor 2, centre O. Each image point lies twice as far from O along the same ray.

$$(x, y) \mapsto (2x, 2y)$$

$$A(2, 3) \mapsto A'(4, 6)$$

Side lengths double. Corresponding angles stay equal, so the shapes are **similar**; they are not congruent here because their sizes differ.

Translations, reflections and rotations preserve size and produce congruent shapes. For an enlargement, always state **both the centre and scale factor**.

