

Circle tangents and graph translations

Topic cliff notes / objectives, key ideas and examples

Learning objectives

- Find a tangent to a circle using a perpendicular radius.
- Interpret horizontal and vertical graph translations.

Key ideas

- A circle centred at the origin has radius given by the square root.

$$x^2 + y^2 = r^2$$

- For two non-vertical perpendicular lines, gradients are negative reciprocals.

$$m_1 m_2 = -1$$

- A horizontal radius has a vertical tangent; a vertical radius has a horizontal tangent.
- The inner subtraction gives a horizontal shift; the outer addition gives a vertical shift.

$$y = f(x - a) + b \quad \text{shifts by} \quad \begin{pmatrix} a \\ b \end{pmatrix}$$

Worked example

Find the tangent to the circle at the given point.

$$x^2 + y^2 = 25, \quad P = (3, 4)$$

$$3^2 + 4^2 = 25, \quad m_{OP} = \frac{4}{3}, \quad m_{\text{tangent}} = -\frac{3}{4}$$

$$y - 4 = -\frac{3}{4}(x - 3)$$

Answer

$$3x + 4y = 25$$

Quick example

Find the turning point of the translated quadratic.

$$y = (x + 2)^2 - 3, \quad x + 2 = 0 \Rightarrow x = -2$$

Answer

$$(-2, -3)$$

Watch out

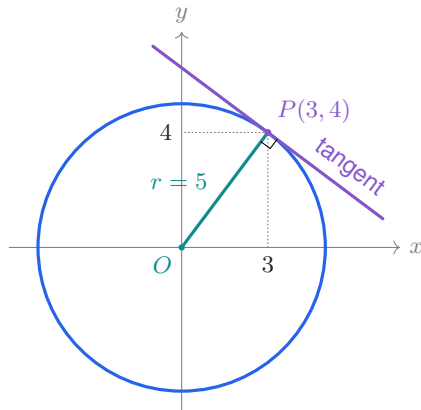
Horizontal and vertical lines need their special tangent cases. Read the horizontal translation with the opposite sign inside the bracket.



Circles and translations

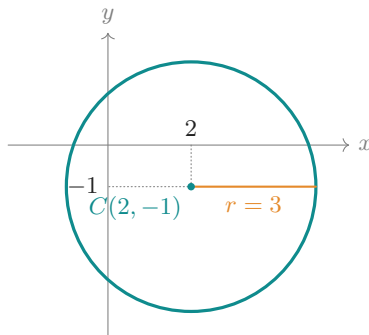
Visual guide / radii, tangents and shifted graphs

A radius meets its tangent at 90°



The point is on the circle because $3^2 + 4^2 = 25$.

A translated circle



$$(x - 2)^2 + (y + 1)^2 = 9$$

Read the centre from $(x - h)^2 + (y - k)^2 = r^2$: here $h = 2$, $k = -1$ and $r = 3$.

Watch out: $(y + 1)^2$ gives centre coordinate -1 , not $+1$. Read both the equation and the labelled graph.

Find the tangent at $P(3, 4)$

For $x^2 + y^2 = 25$, the centre is $O(0, 0)$ and the radius is 5.

$$m_{OP} = \frac{4}{3}, \quad m_{\text{tangent}} = -\frac{3}{4}.$$

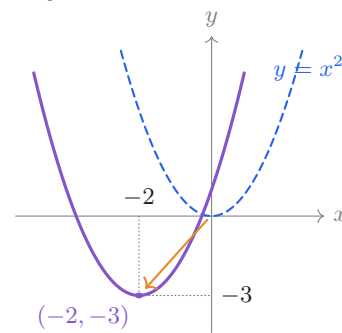
Use the point P in the line equation:

$$y - 4 = -\frac{3}{4}(x - 3) \implies 3x + 4y = 25.$$

Perpendicular gradients multiply to -1 .

At $(5, 0)$ the tangent is $x = 5$; at $(0, 5)$ it is $y = 5$. These vertical or horizontal cases do not need a gradient fraction.

A translated quadratic



$$y = (x + 2)^2 - 3$$

The original turning point $(0, 0)$ moves 2 left and 3 down. Inside $x + 2$ means a left shift.

