

Circle theorems and angle reasons

Topic cliff notes / objectives, key ideas and examples

Learning objectives

- Choose a circle theorem and give a reason for an angle calculation.
- Use radii and tangents to solve a connected angle problem.

Key ideas

- The centre angle is twice the circumference angle subtended by the same arc.

$$\theta_{\text{centre}} = 2\theta_{\text{circumference}}$$

- Angles in the same segment are equal; an angle subtended by a diameter is a right angle.

$$\theta_{\text{semicircle}} = 90^\circ$$

- Opposite angles of a cyclic quadrilateral sum to a straight angle.

$$A + C = 180^\circ$$

- A tangent is perpendicular to its radius. Tangents from the same external point have equal lengths.
- The tangent-chord angle equals the angle subtended by that chord in the opposite segment.

Worked example

Tangents from T touch at A and B; O is the centre. The angle at T is 56 degrees. Find the non-reflex centre angle. A radius meets its tangent at a right angle.

$$\angle OAT = \angle OBT = 90^\circ$$

$$\angle ATB = 56^\circ$$

Angles in the quadrilateral sum to a full turn.

$$\angle AOB = 360^\circ - 90^\circ - 90^\circ - 56^\circ$$

Answer

$$\angle AOB = 124^\circ$$

Quick example

A centre angle is 130 degrees. Find the circumference angle subtended by the same arc.

$$\theta_{\text{circumference}} = \frac{130^\circ}{2}$$

Answer

$$65^\circ$$

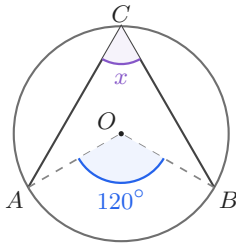
Watch out

Match the correct arc or chord before choosing a theorem. An angle in a semicircle is a right angle, not a straight angle.



Circle theorems: angles in a circle

Blue values are given; magenta letters are unknown. Diagrams are not to scale.



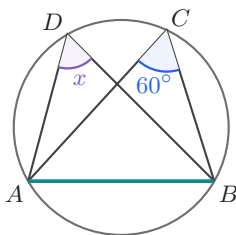
1. The centre angle is twice the circumference angle

The two angles stand on the **same arc** AB . Here O is the centre and C is on the other arc.

$$\angle AOB = 2\angle ACB$$

$$x = 120^\circ \div 2 = \boxed{60^\circ}$$

Reason to write: angle at the centre is twice the angle at the circumference.



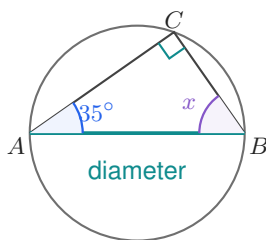
2. Angles in the same segment are equal

Both angles stand on chord AB , with C and D on the **same side** of that chord.

$$\angle ADB = \angle ACB$$

$$x = \boxed{60^\circ}$$

Why it works: both circumference angles are half the angle at the centre on the same arc.



3. The angle in a semicircle is a right angle

AB is a **diameter**, so $\angle ACB = 90^\circ$.

$$x = 180^\circ - 90^\circ - 35^\circ$$

$$x = \boxed{55^\circ}$$

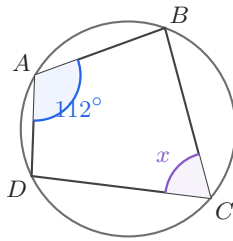
Two reasons: angle in a semicircle; angles in a triangle sum to 180° .

The centre angle on a diameter is 180° ; half of it is 90° .



Circle theorems: lines and quadrilaterals

Look for a cyclic shape, a tangent or a perpendicular. Where shown, O is the centre.



4. Opposite angles of a cyclic quadrilateral add to 180°

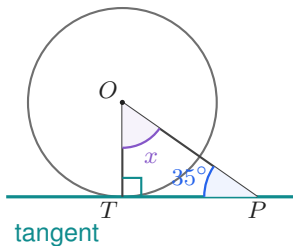
A quadrilateral is **cyclic** when all four vertices lie on one circle. A and C are opposite vertices.

$$x + 112^\circ = 180^\circ$$

$$x = \boxed{68^\circ}$$

Reason to write: opposite angles in a cyclic quadrilateral sum to 180° .

Adjacent angles need not sum to 180° .



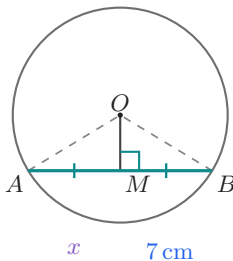
5. A tangent is perpendicular to the radius

The line touches the circle at T . Radius OT meets the tangent at a **right angle**.

$$\angle OTP = 90^\circ$$

$$x = 180^\circ - 90^\circ - 35^\circ = \boxed{55^\circ}$$

Reason to write: radius and tangent are perpendicular at the point of contact.



6. A perpendicular from the centre bisects a chord

OM is perpendicular to chord AB , so M is its **midpoint**.

$$AM = MB$$

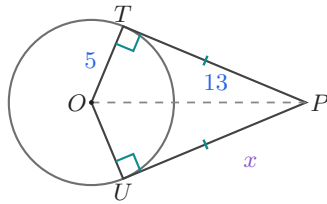
$$x = \boxed{7 \text{ cm}} \quad AB = 14 \text{ cm}$$

Why it works: the right triangles share OM and have equal radii $OA = OB$, so they are congruent.



Circle theorems: tangents and reason chains

Name the chord or arc you use. State a reason at each step.



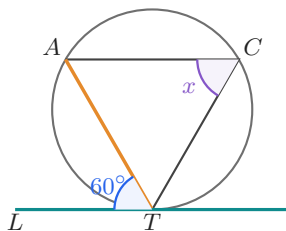
7. Tangents from one external point are equal

All lengths are in cm. The two tangents from P have equal lengths: $PT = PU$.

$$PT^2 = OP^2 - OT^2 = 13^2 - 5^2 = 144$$

$$PT = 12 \quad x = PU = \boxed{12 \text{ cm}}$$

Reasons: radius is perpendicular to tangent; Pythagoras; tangents from the same point are equal.



8. The alternate segment theorem

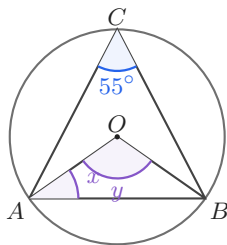
The angle between tangent TL and chord TA equals the angle at C in the **opposite segment**.

$$\angle ATL = \angle TCA$$

$$x = \boxed{60^\circ}$$

Reason to write: alternate segment theorem.

Follow the highlighted chord TA to its opposite angle; do not choose the angle at A .



A short reason chain: circle theorem + isosceles triangle

First use the same arc AB :

$$y = 2 \times 55^\circ = 110^\circ$$

Radii $OA = OB$, so triangle OAB is isosceles. Its two base angles are equal.

$$x = \frac{180^\circ - 110^\circ}{2} = \boxed{35^\circ}$$

Reasons: centre/circumference theorem, equal radii, then triangle angle sum.

