

Conditional probability and dependent trees

Topic cliff notes / objectives, key ideas and examples

Learning objectives

- Calculate dependent probabilities for draws without replacement.
- Use the condition to identify the correct denominator group.

Key ideas

- Without replacement, reduce the total and the count of the colour removed.
- Multiply probabilities along a path; add mutually exclusive paths for the requested outcome.
- Conditioning restricts the sample space to the stated group.

$$P(A | B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

- In a two-way table, divide by the total of the group specified by the condition.

Worked example

A bag has four red and three blue counters. Draw two at random without replacement. Find the probability of exactly one of each colour.

$$P(RB) = \frac{4}{7} \times \frac{3}{6} = \frac{2}{7}$$

$$P(BR) = \frac{3}{7} \times \frac{4}{6} = \frac{2}{7}$$

$$P(\text{one of each}) = P(RB) + P(BR)$$

Answer

$$\frac{2}{7} + \frac{2}{7} = \frac{4}{7}$$

Quick example

In the same bag, find the probability the second counter is red given the first is red.

remaining counters : 3 red out of 6

Answer

$$P(R_2 | R_1) = \frac{3}{6} = \frac{1}{2}$$

Watch out

The second draw has a smaller total without replacement. Include both possible orders when finding exactly one of each colour.

