

Iteration and numerical solutions

Topic cliff notes / objectives, key ideas and examples

Learning objectives

- Use a starting value to generate successive estimates.
- Recognise a fixed point and keep unrounded intermediate values.

Key ideas

- The next estimate is calculated from the latest estimate.

$$x_{n+1} = f(x_n), \quad x_0 = \text{starting value}$$

- Keep full calculator values between iterations; round only the reported answers.
- A fixed point satisfies this equation; iteration does not always converge.

$$x = f(x)$$

- A sign change for a continuous function guarantees at least one root between the two inputs.

Worked example

Start at two and use the recurrence. Report the first three estimates to three decimal places.

$$x_{n+1} = \sqrt{x_n + 4}, \quad x_0 = 2$$

$$x_1 = \sqrt{6} = 2.449489742 \dots$$

$$x_2 = \sqrt{x_1 + 4} = 2.539584561 \dots$$

$$x_3 = \sqrt{x_2 + 4} = 2.557261144 \dots$$

Answer

$$x_1 \approx 2.449, \quad x_2 \approx 2.540, \quad x_3 \approx 2.557$$

Quick example

Start at four and calculate the second estimate.

$$x_{n+1} = 0.5x_n + 3, \quad x_0 = 4 \quad \Rightarrow \quad x_1 = 5$$

Answer

$$x_2 = 0.5(5) + 3 = 5.5$$

Watch out

Use the full latest estimate as the next input, not its rounded display value. Check whether the values actually settle.

