

Non-right-angled trigonometry

Topic cliff notes / objectives, key ideas and examples

Learning objectives

- Select the sine rule, cosine rule or triangle area formula.
- Match opposite sides and angles, and round final values only.

Key ideas

- Use opposite side-angle pairs in the sine rule.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- The cosine rule uses the included angle between the two known sides.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

- The area formula uses the included angle.

$$\text{area} = \frac{1}{2}ab \sin C$$

- Use degree mode for degree angles. A sine-rule angle may have two possible values; check the triangle context.

Worked example

Sides of 7 cm and 9 cm enclose 60 degrees. Find the third side and area to three significant figures.

$$c^2 = 7^2 + 9^2 - 2(7)(9) \cos 60^\circ = 67$$

$$c = \sqrt{67} \text{ cm}$$

$$\text{area} = \frac{1}{2}(7)(9) \sin 60^\circ = \frac{63\sqrt{3}}{4} \text{ cm}^2$$

Answer

$$c \approx 8.19 \text{ cm}, \quad \text{area} \approx 27.3 \text{ cm}^2$$

Quick example

An 8 cm side is opposite 30 degrees. Find the side opposite 60 degrees exactly.

$$\frac{8}{\sin 30^\circ} = \frac{b}{\sin 60^\circ}$$

Answer

$$b = \frac{8 \sin 60^\circ}{\sin 30^\circ} = 8\sqrt{3} \text{ cm}$$

Watch out

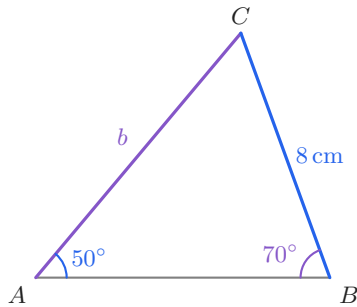
Pythagoras requires a right angle. Match each sine-rule side to its opposite angle; use the included angle for the cosine rule.



Trigonometry: choose the rule

Visual guide / opposite pairs and included angles

Sine rule: use opposite pairs

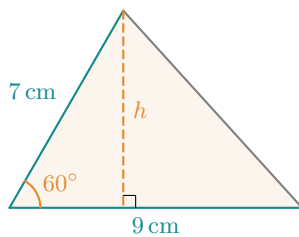


The 8 cm side is opposite 50° ; b is opposite 70° .

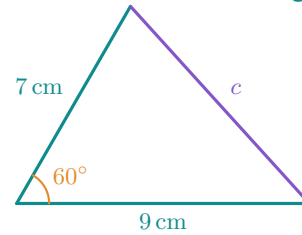
$$\frac{b}{\sin 70^\circ} = \frac{8}{\sin 50^\circ}$$

$$b = \frac{8 \sin 70^\circ}{\sin 50^\circ} \approx \boxed{9.81 \text{ cm}}.$$

Triangle area: the same included angle



Cosine rule: two sides and their angle



The 60° angle lies *between* the 7 cm and 9 cm sides.

$$c^2 = 7^2 + 9^2 - 2(7)(9) \cos 60^\circ$$

$$c^2 = 67, \quad c = \sqrt{67} \approx \boxed{8.19 \text{ cm}}.$$

The perpendicular height is $h = 7 \sin 60^\circ$. Thus

$$\text{area} = \frac{1}{2} \times 9 \times h = \frac{1}{2}(7)(9) \sin 60^\circ.$$

$$\text{area} = \frac{63\sqrt{3}}{4} \approx \boxed{27.3 \text{ cm}^2}.$$

Keep the full calculator value until your final rounding.

Choose quickly

Opposite side–angle pair known: sine rule.

Two sides and the included angle: cosine rule for a third side.

Two sides and the included angle: $\frac{1}{2}ab \sin C$ for area.

Watch out: use degree mode. A sine-rule angle can have two possibilities; use the triangle's other information to decide.

