

Quadratic sequences

Topic cliff notes / objectives, key ideas and examples

Learning objectives

- Find a quadratic rule from first and second differences.
- Use the rule to calculate a term at an integer position.

Key ideas

- First differences compare consecutive terms; second differences compare consecutive first differences.
- A quadratic sequence has a constant second difference twice its quadratic coefficient.

$$T_n = an^2 + bn + c, \quad \text{second difference} = 2a$$

- Subtract the square part; find a linear rule for the remainder.
- The first term is position one unless another start is specified.

$$n \in \{1, 2, 3, \dots\}$$

Worked example

Find the rule for the sequence.

$$3, 8, 15, 24, 35, \dots$$

$$\text{first differences : } 5, 7, 9, 11; \quad \text{second differences : } 2, 2, 2$$

Subtract the square-number sequence to obtain a linear remainder.

$$a = \frac{2}{2} = 1; \quad \text{remainder : } 2, 4, 6, 8, 10, \dots$$

$$T_n - n^2 = 2n$$

Answer

$$T_n = n^2 + 2n$$

Quick example

Find the fourth term of the quadratic sequence.

$$T_n = 2n^2 - n, \quad T_4 = 2(4^2) - 4$$

Answer

$$T_4 = 28$$

Watch out

The quadratic coefficient is half the second difference. Check the complete rule against several original terms.

