

Quadratics: formula and completing the square

Topic cliff notes / objectives, key ideas and examples

Learning objectives

- Solve quadratics with the quadratic formula.
- Complete the square and identify a turning point.

Key ideas

- Write the equation in standard form; keep coefficient signs.

$$ax^2 + bx + c = 0, \quad a \neq 0$$

- The formula gives both possible real roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Positive, zero or negative discriminants give two, one repeated or no real roots.

$$D = b^2 - 4ac$$

- Halve the linear coefficient, then correct the constant.

$$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

- The completed-square form shows the turning point.

$$y = a(x - p)^2 + q \Rightarrow (p, q)$$

Worked example

Solve using the quadratic formula.

$$2x^2 + 3x - 2 = 0 \Rightarrow a = 2, b = 3, c = -2$$

$$D = 3^2 - 4(2)(-2) = 25$$

$$x = \frac{-3 \pm \sqrt{25}}{4} = \frac{-3 \pm 5}{4}$$

Answer

$$x = \frac{1}{2} \quad \text{or} \quad x = -2$$

Quick example

Complete the square.

$$x^2 - 8x + 7 = (x - 4)^2 - 16 + 7$$

Answer

$$(x - 4)^2 - 9$$

Watch out

Divide the whole numerator by the denominator. Keep both signs when taking a square root to solve an equation.

