

Similarity: lengths, areas and volumes

Topic cliff notes / objectives, key ideas and examples

Learning objectives

- Convert a length scale factor into area and volume factors.
- Work backwards from an area or volume ratio to a length ratio.

Key ideas

- For similar shapes, use corresponding measurements and a consistent comparison direction.
- For a positive length scale factor, square it for area and cube it for volume.

$$k > 0, \quad \text{area factor} = k^2, \quad \text{volume factor} = k^3$$

- Take square roots of an area ratio to recover the length ratio.

$$a : b \Rightarrow \sqrt{a} : \sqrt{b}$$

- Take cube roots of a volume ratio to recover the length ratio.

$$a : b \Rightarrow \sqrt[3]{a} : \sqrt[3]{b}$$

Worked example

In two separate similarity examples, the small-to-large length ratio is 2 to 3. One small shape has area 24 square centimetres; a different small solid has volume 80 cubic centimetres. Find the corresponding large area and volume.

$$k = \frac{3}{2}$$

$$A_{\text{large}} = 24 \left(\frac{3}{2} \right)^2$$

$$V_{\text{large}} = 80 \left(\frac{3}{2} \right)^3$$

Answer

$$A_{\text{large}} = 54 \text{ cm}^2, \quad V_{\text{large}} = 270 \text{ cm}^3$$

Quick example

Similar solids have volume ratio 27 to 64. Find the length ratio.

$$\sqrt[3]{27} : \sqrt[3]{64}$$

Answer

$$3 : 4$$

Watch out

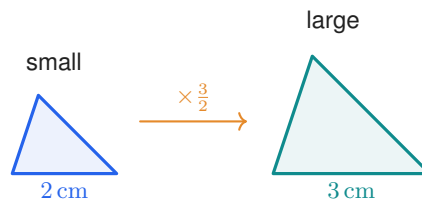
Do not apply the length factor directly to areas or volumes. Keep small-to-large and large-to-small comparisons consistent.



Similarity: what scales?

Visual guide / length, area and volume factors

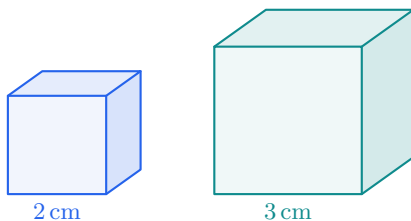
Similar triangles: each length is multiplied by $\frac{3}{2}$



Corresponding angles stay equal. The large triangle is the small triangle enlarged: every corresponding length has the same scale factor.

$$\text{small : large lengths} = 2 : 3, \quad \text{areas} = 2^2 : 3^2 = \boxed{4 : 9}.$$

Similar solids: three dimensions scale



The two cube drawings use the same perspective and scale. Their volumes are 8 and 27 cm³.

For any similar solids with length ratio 2 : 3:

$$\text{area ratio} = \boxed{4 : 9}, \quad \text{volume ratio} = \boxed{8 : 27}.$$

Surface-area example: a small solid has surface area 24 cm².

$$A_{\text{large}} = 24 \left(\frac{3}{2} \right)^2 = \boxed{54 \text{ cm}^2}.$$

Separate volume example: a small solid has volume 80 cm³.

$$V_{\text{large}} = 80 \left(\frac{3}{2} \right)^3 = \boxed{270 \text{ cm}^3}.$$

Working backwards

An area ratio 9 : 16 gives length ratio $\sqrt{9} : \sqrt{16} = 3 : 4$.

A volume ratio 27 : 64 gives length ratio $\sqrt[3]{27} : \sqrt[3]{64} = 3 : 4$.

Watch out: choose a direction first. Small to large uses $k = \frac{3}{2}$; large to small uses $k = \frac{2}{3}$. Square k for area and cube it for volume.

