

Simultaneous linear and quadratic equations

Topic cliff notes / objectives, key ideas and examples

Learning objectives

- Substitute a linear expression into a quadratic equation.
- Find and check every matching coordinate pair.

Key ideas

- At an intersection, both equations hold for the same coordinate pair.
- Substitute the whole expression, using brackets whenever it is squared.
- Solve the resulting quadratic; there may be two, one or no real solutions.
- Recover the matching second coordinate for every first coordinate. Check both equations.

Worked example

Solve the line and quadratic simultaneously.

$$y = x + 1, \quad y = x^2 - 1$$

$$x + 1 = x^2 - 1 \Rightarrow (x - 2)(x + 1) = 0$$

$$x = 2 \Rightarrow y = 3, \quad x = -1 \Rightarrow y = 0$$

Answer

$$(x, y) = (2, 3) \quad \text{or} \quad (-1, 0)$$

Quick example

Solve the horizontal line and circle simultaneously.

$$y = 3, \quad x^2 + y^2 = 25 \Rightarrow x^2 = 16$$

$$x = \pm 4$$

Answer

$$(x, y) = (4, 3) \quad \text{or} \quad (-4, 3)$$

Watch out

Do not mix coordinates from different solutions. A positive square can come from either a positive or negative number.

