

Surds and fractional indices

Topic cliff notes / objectives, key ideas and examples

Learning objectives

- Simplify and combine surds while keeping exact values.
- Evaluate fractional and negative indices.

Key ideas

- For positive bases, a fractional index means root, then power.

$$a^{m/n} = (\sqrt[n]{a})^m \quad (a > 0; m \in \mathbb{Z}, n \in \mathbb{Z}_{>0})$$

- For a non-zero base and positive integer power, a negative index gives a reciprocal.

$$a^{-k} = \frac{1}{a^k} \quad (a \neq 0, k \in \mathbb{Z}_{>0})$$

- Take square factors outside the root. Collect only like surds.

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \quad (a, b \geq 0)$$

- Rationalise a single surd denominator by multiplying both parts by that surd.

$$\frac{1}{\sqrt{a}} = \frac{\sqrt{a}}{a} \quad (a > 0)$$

- For a two-term surd denominator, multiply both parts by its non-zero conjugate.

Worked example

Simplify the surd expression exactly.

$$\sqrt{50} + 3\sqrt{8}$$

Extract the square factors.

$$\sqrt{50} = 5\sqrt{2}, \quad \sqrt{8} = 2\sqrt{2}$$

$$5\sqrt{2} + 3(2\sqrt{2}) = (5 + 6)\sqrt{2}$$

Answer

$$11\sqrt{2}$$

Quick example

Evaluate the fractional power.

$$27^{2/3} = \left(\sqrt[3]{27}\right)^2 = 3^2$$

Answer

$$9$$

Watch out

A root of a sum is not usually the sum of the roots. A negative index does not make the answer negative.

